Introduction to Algebra

Variables

A variable is a symbol that represents a number. Usually we use letters such as n, t, or x for variables. For example, we might say that s stands for the side-length of a square. We now treat s as if it were a number we could use. The perimeter of the square is given by $4 \times s$. The area of the square is given by $s \times s$. When working with variables, it can be helpful to use a letter that will remind you of what the variable stands for: let n be the number of people in a movie theater; let t be the time it takes to travel somewhere; let d be the distance from my house to the park.

Expressions

An expression is a mathematical statement that may use numbers, variables, or both.

Example:

The following are examples of expressions:

2

x

$3 + 7$

$2 \times y + 5$

$2 + 6 \times (4 - 2)$

$z + 3 \times (8 - z)$

Example:

Roland weighs 70 kilograms, and Mark weighs k kilograms. Write an expression for their combined weight. The combined weight in kilograms of these two people is the sum of their weights, which is $70 + k$.

Example:

A car travels down the freeway at 55 kilometers per hour. Write an expression for the distance the car will have traveled after h hours. Distance equals rate times time, so the distance traveled is equal to $55 \times h$..

Example:

There are 2000 liters of water in a swimming pool. Water is filling the pool at the rate of 100 liters per minute. Write an expression for the amount of water, in liters, in the swimming pool after m minutes. The amount of water added to the pool after m minutes will be 100 liters per minute times m, or $100 \times m$. Since we started with 2000 liters of water in the pool, we add this to the amount of water added to the pool to get the expression $100 \times m + 2000$.

To evaluate an expression at some number means we replace a variable in an expression with the number, and simplify the expression.

Example:

Evaluate the expression $4 \times z + 12$ when $z = 15$.

We replace each occurrence of z with the number 15, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$4 \times z + 12$ becomes

$4 \times 15 + 12 =$

$60 + 12 =$

Example:

Evaluate the expression $(1 + z) \times 2 + 12 \div 3 - z$ when $z = 4$.

We replace each occurrence of z with the number 4, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$(1 + z) \times 2 + 12 \div 3 - z$ becomes

$(1 + 4) \times 2 + 12 \div 3 - 4 =$

$5 \times 2 + 12 \div 3 - 4 =$

$10 + 4 - 4 =$

10.

Equations

An equation is a statement that two numbers or expressions are equal. Equations are useful for relating variables and numbers. Many word problems can easily be written down as equations with a little practice. Many simple rules exist for simplifying equations.

Example:

The following are examples of equations:

$2 = 2$

$17 = 2 + 15$

$x = 7$

$7 = x$

$t + 3 = 8$

$3 \times n + 12 = 100$

$w + 4 = 12 - w$

$y - 1 - 2 - 9.3 = 34$

$3 \times (d + 4) - 11 = 321 - 2^3$

Example:

Translate the following word problem into an equation:

My age in years *y* plus 20 is equal to four times my age, minus 10.

The first expression stands for "my age in years plus 20", which is $y + 20$.

This is equal to the second expression for "four times my age, minus 10", which is $4 \times y - 10$.

Setting these two expressions equal to one another gives us the equation:

$y + 20 = 4 \times y - 10$

Solution of an Equation

When an equation has a variable, the solution to the equation is the number that makes the equation true when we replace the variable with its value.

Example:

We say $y = 3$ is a solution to the equation $4 \times y + 7 = 19$, for replacing each occurrence of *y* with 3 gives us

$4 \times 3 + 7 = 19$ ==>

$12 + 7 = 19$ ==>

$19 = 19$ which is true.

Examples:

$x = 100$ is a solution to the equation $x \div 2 - 40 = 10$

$z = 12$ is a solution to the equation $5 \times (z - 6) = 30$

Counterexample:

$y = 10$ is NOT a solution to the equation $4 \times y + 7 = 19$. When we replace each y with 10, we get

$4 \times 10 + 7 = 19$ ==>

$40 + 7 = 19$ ==>

$47 = 19$ not true!

Counterexamples:

$x = 200$ is NOT a solution to the equation $x \div 2 - 40 = 10$

$z = 20$ is NOT a solution to the equation $5 \times (z - 6) = 30$

Simplifying Equations

To find a solution for an equation, we can use the basic rules of simplifying equations. These are as follows:

1) You may evaluate any parentheses, exponents, multiplications, divisions, additions, and subtractions in the usual order of operations. When evaluating expressions, be careful to use the associative and distributive properties properly.

2) You may combine like terms. This means adding or subtracting variables of the same kind. The expression $2x + 4x$ simplifies to $6x$. The expression $13 - 7 + 3$ simplifies to 9.

3) You may add any value to both sides of the equation.

4) You may subtract any value from both sides of the equation. This is best done by adding a negative value to each side of the equation.

5) You may multiply both sides of the equation by any number except 0.

6) You may divide both sides of the equation by any number except 0.

Hint: Since subtracting any number is the same as adding its negative, it can be helpful to replace subtractions with additions of a negative number.

Example:

This problem illustrates grouping like terms and dealing with subtraction in an equation.

Solve $x - 12 + 20 = 37$.

Replacing the -12 with a +(-12), we get

$x + (-12) + 20 = 37$.

Since addition is associative, the two like terms (the integers) may be combined.

(12) + 20 = 8

The left side of the equation becomes

x + 8 = 37.

Now we may subtract 8 from each side of the equation, (we will actually add a -8 to each side).

$x + 8 + (-8) = 37 + (-8)$

$x + 0 = 29$

$x = 29$

We can check this solution in the original equation:

29 - 12 + 20 = 37x + 0 = 29

17 + 20 = 37

37 = 37 so our solution is correct.

Example:

This problem illustrates the proper use of the distributive property.

Solve $2 \times (x + 1 + 4) = 20$.

Grouping like terms in the parentheses, the left side of the equation becomes

$2 \times (x + 1 + 4)$ ==> $2 \times (x + 5)$.

Using the distributive property,

$2 \times (x + 5)$ ==> $2 \times x + 2 \times 5$.

Carrying out multiplications,

$2 \times x + 2 \times 5$ ==> to $2x + 10$.

The equation now becomes

$2x + 10 = 20$.

Subtracting a 10 (adding a -10) to each side gives us

$2x + 10 + (-10) = 20 + (-10)$ ==>

$2x + (10 + (-10)) = 20 - 10$ ==>

$2x + 0 = 10$ ==>

$2x = 10$.

Since the x is multiplied by 2, we divide both sides by 2 to solve for x:

$2x = 10$ ==>

$2x \div 2 = 10 \div 2$ ==>

$(2x)/2 = 5$ ==>

$x = 5$.

We can check this solution in the original equation:

$2 \times (5 + 1 + 4) = 20$ ==>

$2 \times 10 = 20$ ==>

$20 = 20$ so our solution is correct.

Combining like terms

One of the most common ways to simplify an expression is to combine like terms. Numeric terms may be combined, and any terms with the same variable part may be combined.

Example:

Consider the expression $2 + 7x + 12 - 3x - 5$. The numeric like terms are the numbers 2, 12, and 5. The variable like terms are $7x$ and $3x$. Combining the numeric like terms, we have $2 + 12 - 5 = 14 - 5 = 9$. Combining the variable like terms, we have $7x - 3x = 4x$, so the expression $2 + 7x + 12 - 3x - 5$ simplifies to $9 + 4x$.

Simplifying with addition and subtraction

We can use addition and subtraction to get all the terms with variables on one side of an equation, and all the numeric terms on the other.

The equations $3x = 17$, $21 = y$, and $z/12 = 24$ each have a variable term on one side of the = sign, and a number on the other.

The equations $x + 3 = 12$, $21 = 30 - y$, and $(z + 2) \times 4 = 10$ do not.

We usually do this after simplifying each side using the distributive rules, eliminating parentheses, and combining like terms. Since addition is associative, it can be helpful to add a negative number to each side instead of subtracting to avoid mistakes.

Examples:

For the equation $3x + 4 = 12$, we can isolate the variable term on the left by subtracting a 4 from both sides:

$3x + 4 - 4 = 12 - 4$ ==>

$3x = 8$.

For the equation $7y - 200 = 10$, subtracting the 200 on the left side is the same as adding a -200:

$7y + (-200) = 10$.

If we add 200 to both sides of the equation, the 200 and -200 will cancel each other:

$7y + (-200) + 200 = 10 + 200$ ==>

$7y = 210$.

For the equation $8 = 20 - z$, we can add z to both sides to get $8 + z = 20 - z + z$ ==> $8 + z = 20$. Now subtracting 8 from both sides,

$8 + z - 8 = 20 - 8$ ==>

$z = 12$, so we get a solution for z.

Simplfying by multiplication

When solving for a variable, we want to get a solution like $x = 3$ or $z = 2001$. When a variable is divided by some number, we can use multiplication on both sides to solve for the variable.

Example:

Solve for x in the equation $x \div 12 = 5$.

Since the x on the left side is being divided by 12, the equation is the same as $x \times 1/12 = 5$. Multiplying both sides by 12 will cancel the 1/12 on the left side:

$x \times 1/12 \times 12 = 5 \times 12$ ==>

$x \times 1 = 60$ ==>

$x = 60$.

Simplifying by division

When solving for a variable, we want to get a solution like $x = 3$ or $z = 2001$. When a variable is multiplied by some number, we can use division on both sides to solve for the variable.

Example:

Solve for x in the equation $7x = 133$. Since the x on the left side is being multiplied by 7, we can divide both sides by 7 to solve for x:

$7x \div 7 = 133 \div 7$ ==>

$(7x)/7 = 133 \div 7$ ==>

$x/1 = 19$ ==>

$x = 19$.

Note that dividing by 7 is the same as multiplying both sides by 1/7.

Word problems as equations

When converting word problems to equations, certain "key" words tell you what kind of operations to use: addition, multiplication, subtraction, and division. The table below shows some common phrases and the operation to use.

Word	**Operation**	**Example**	**As an equation**
sum	addition	The sum of my age and 10 equals 27.	$y + 10 = 27$
difference	subtraction	The difference between my age and my younger sister's age, who is 11 years old, is 5 years.	$y - 11 = 5$
product	multiplication	The product of my age and 14 is 168.	$y \times 14 = 168$
times	multiplication	Three times my age is 60.	$3 \times y = 60$
less than	subtraction	Seven less than my age equals 32.	$y - 7 = 32$
total	addition	The total of my pocket change and 20 dollars is $22.43.	$y + 20 = 22.43$
more than	addition	Eleven more than my age equals 43.	$11 + y = 43$

Sequences

A sequence is a list of items. We can specify any item in the list by its place in the list: first, second, third, fourth, and so on. Many useful lists have patterns so we know what items occur in each place in the list. There are 2 kinds of sequences. A finite sequence is a list made up of a finite number of items. An infinite sequence is a list that continues without end.

Examples:

The following are examples of finite sequences.

The sequence 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 is the sequence of the first 10 odd numbers.

The sequence *a, e, i, o, u,* is the sequence of vowels in the alphabet.

The sequence *m, m, m, m, m, m* is the sequence of 6 *m's.*

The sequence 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0 is the sequence of 12 alternating 1's and 0's.

The sequence 1, 2, 3, 4, ..., 9998, 9999, 10000 is the sequence of the first ten thousand integers.

The sequence 0, 1, 4, 9, 16, 25, 36, 49 is the sequence of the squares of the first 8 whole numbers.

Examples:

The following are examples of infinite sequences.

The sequence 2, 4, 6, 8, 10, 12, 14, 16, ... is the sequence of even whole numbers. The 100th place in this sequence is the number 200.

The sequence *a, b, c, a, b, c, a, b, c, a, b,* ... is the sequence of the letters a, b, c, repeating in this pattern forever.

The 100th place in this sequence is the letter *a.* The 300th place in this sequence is the letter *c.*

The sequence -1, 2, -3, 4, -5, 6, -7, 8, -9, ... is the sequence of integers with alternating signs. The 10th place in this sequence is 10. The 100th place in this sequence is 100. The 101st place in this sequence is -101.

The sequence 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, ... is a sequence of 1's separated by 1 zero, then 2 zeros, then 3 zeros, and so on. The 100th place in this sequence is a 0. The 105th place in this sequence is a 1.

The sequence 1, 3, 6, 10, 15, 21, 28, 36, 45, ... is the sequence of places the 1 occurs in the sequence of 1's and 0's above! If this sequence seems strange, note the difference between pairs of numbers next to one another:

3 - 1 = 2

6 - 3 = 3

10 - 6 = 4

15 - 10 = 5

21 - 15 = 6

28 - 21 = 7

Checking these differences makes the pattern clearer.

1, 1, 1, 1, 1, 1, ... is the sequence where every item in the list is the number 1.

1, 2, 3, 4, 5, 6, 7, ... is the sequence of counting numbers. Each item in the list is its place number in the list.

a, b, a, b, a, b, a, b, ... is the sequence of alternating letters a and b. The a's occur in odd-numbered places, and the b's occur in the even-numbered places.

1/1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, ... is the sequence of reciprocals of the whole numbers.

1, 4, 9, 16, 25, 36, 49, 64, 81, ... is the sequence of squares of the whole numbers.

a, e, i, o, u, a, e, i, o, u, a, e, ... is the repeating sequence of vowels in the alphabet.

4, 7, 10, 13, 16, 19, 22, 25, ... is the sequence of numbers beginning with the number 4, and each number in the list is 3 more than the number before it.

Tutorial on Equation of Line

This is a tutorial on how to find the slopes and equations of lines. A review of the main results concerning lines and slopes and then examples with detailed solutions are presented.

Slope of a Line:

If a line passes through two distinct points $P_1(x_1 , y_1)$ and $P_2(x_2, y_2)$, its slope is given by:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

with x_2 not equal to x_1.
This is another interactive tutorial on the slope of a line.

General Equation of a Straight line:

The general equation of straight line is given by:

$$Ax + By = C$$

where A, B and C are constants and A and B cannot be **both** zero. For an interactive exploration of this equation Go here .
Any straight line in a rectangular system has an equation of the form above.

Slope intercept form of a Line:

The equation of a line with a defined slope m can also be written as follows:

$$y = mx + b$$

where m is the slope of the line and b is the y intercept of the graph of the line.
The above form is called the slope intercept form of a line. To understand why, go to this interactive tutorial.

Point-Slope form of a line:

An equation of a line through a point $P(x_1 , y_1)$ with slope m is given by

$$y - y_1 = m(x - x_1)$$

Vertical and Horizontal lines:

a - If we set A to zero in the general equation, we obtain an equation in y only of the form

$$By = C$$

which gives y = C/B = k; k is a constant. This is a horizontal line with slope 0 and passes through all points with y coordinate equal to k.

b - If we set B to zero in the general equation, we obtain

$$Ax = C$$

which gives x = C/A = h; h is constant. This is a vertical line with undefined slope and passes through all points with x coordinate equal to h.

Parallel Lines:

Two non vertical lines are parallel if and only if their slopes are equal.

Perpendicular Lines:

Two non vertical lines are perpendicular if and only if their slopes m_1 and m_2 are such that

$$m_1 * m_2 = -1$$

<u>Example 1:</u> Find the slope of a line passing through the points

a. (2 , 3) and (0 , -1)
b. (-2 , 4) and (-2 , 6)
c. (5 , 2) and (-7 , 2)

<u>Solution to Example1:</u>

a. $m = (y_2 - y_1) / (x_2 - x_1) = (-1 - 3) / (0 - 2) = 2$

b. $m = (6 - 4) / (-2 + 2)$
The division by $-2 + 2 = 0$ is undefined and the slope in this case is undefined. The line passing through the given points is a vertical line.

c. $m = (2 - 2) / (-7 - 5) = 0$
The slope is equal to 0 and the line through the given points is a horizontal line.

Matched Exercise 1: Find the slope of a line passing through the points

1. (-2 , 7) and (-2 , -1)
2. (2 , 4) and (-2 , 6)
3. (-1 , -2) and (4 , -2)

Example 2: Find the equation of the line that passes through the point (-2 , 5) and has a slope of -4.

Solution to Example 2:

- Substitute y_1 , x_1 and m in the point slope form of a line
$y - y_1 = m(x - x_1)$

$y - 5 = - 4(x - (-2))$
$y = - 4x - 3$

Matched Exercise 2: Find the equation of the line that passes through the point (3 , 0) and has a slope of - 1.

Example 3: Find the equation of the line that passes through the points (0 , -1) and (3 , 5).

Solution to Example 3:

- We first calculate the slope of the line
$m = (5 - (-1)) / (3 - 0) = 6 / 3 = 2$

- Use the slope and any of the two points to write the equation of

the line using the point slope form.
$y - y_1 = m(x - x_1)$
using the first point
$y - (-1) = 2(x - 0)$
$y = 2x - 1$

Matched Exercise 3: Find the equation of the line that passes through the points (2 , 0) and (3 , 3).

Example 4: Find the slope of the line given by the equation

$$-2x + 4y = 6$$

Solution to Example 4:

- Given the equation
  $-2x + 4y = 6$
- Write the equation in slope intercept form
  $4y = 2x + 6$
  $y = (1/2)x + 3/2$
- The slope of the line is given by the coefficient of x and is equal to 1/2.

Matched Exercise 4: Find the slope of the line given by the equation

$$x - 3y = -9$$

Example 5: Find an equation of the line that passes through the point (-2 , 3) and is parallel to the line $4x + 4y = 8$

Solution to Example 5:

- Let m_1 be the slope of the line whose equation is to be found and m_2 the slope of the given line. Rewrite the given equation in slope intercept form and find its slope.
  $4y = -4x + 8$

- Divide both sides by 4
  $y = -x + 2$
  slope $m_2 = -1$.
- Two lines are parallel if and only if they have equal slopes
  $m_1 = m_2 = -1$
- We now use the point slope form to find the equation of the line with slope m_1.
  $y - 3 = -1(x - (-2))$
  which may be written as
  $y = -x + 1$

Matched Exercise 5: Find an equation of the line that passes through the point (-1 , 0) and is parallel to the line
$-2x + 2y = 8$

Example 6: Find an equation of the line that passes through the point (0 , -3) and is perpendicular to the line $-x + y = 2$.

Solution to Example 6:

- Let m_1 be the slope of the line whose equation is to be found and m_2 the slope of the given line. Rewrite the given equation in slope intercept form and find its slope.
  $y = x + 2$
  slope $m_2 = 1$
- Two lines are perpendicular if and only their slopes are such that
  $m_1 * m_2 = -1$
- This gives m1 = -1
- We now use the point slope form to find the equation of the line with slope m_1.
  $y - (-3) = -1(x - 0)$
  which may be written
  $y = -x - 3$

Matched Exercise 6: Find an equation of the line that passes through the point (-2 , 1) and is perpendicular to the line x + 2y = -2.

More references on lines and slopes.

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1. Easy to use calculator to find slope and equation of a line through two points.Find Distance, Slope and Equation of Line - Calculator.
2. Another calculator to find **slope**, x and y intercepts given the equation of a line.Find Slope and Intercepts of a Line - Calculator
3. Find Distance From a Point to a Line - Calculator
4. Find a Parallel Line Through a Point: Find a line that is parallel to another line and passes through a point.
5. Find a Perpendicular Line Through a Point: Find a line that is perpendicular to another line and passes through a point.
6. General Equation of a Line: ax + by = c - Applet
7. Slope Intercept Form Of a Line
8. Slope of a Line

Plotting Linear Graphs

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a straight line graph we need to find the coordinates of *at least two points* that fit the rule.

Example 6

Plot the graph of $y = 3x + 2$.

Solution:

Construct a table and choose simple x values.

x	-2	-1	0	1	2
y					

In order to find the y values for the table, substitute each x value into the rule $y = 3x + 2$.

$$\text{When } x = -2,\ y = 3(-2) + 2 \\ = -6 + 2 \\ = -4$$

$$\text{When } x = -1,\ y = 3(-1) + 2 \\ = -3 + 2 \\ = -1$$

$$\text{When } x = 0,\ y = 3 \times 0 + 2 \\ = 0 + 2 \\ = 2$$

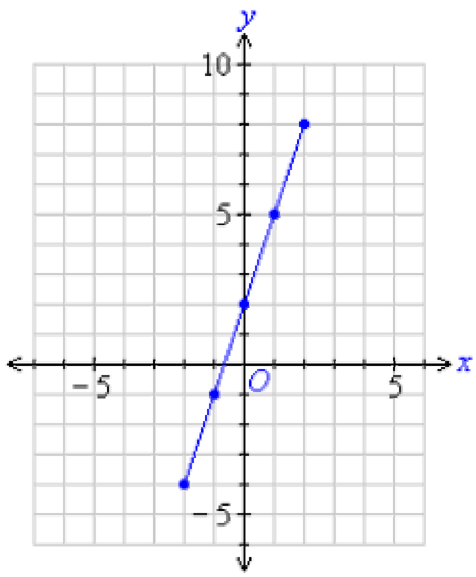
$$\text{When } x = 1,\ y = 3 \times 1 + 2 \\ = 3 + 2 \\ = 5$$

$$\text{When } x = 2,\ y = 3 \times 2 + 2 \\ = 6 + 2 \\ = 8$$

The table of values obtained after entering the values of y is as follows:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

$y = 3x + 2$

When $x = -2$, $y = 3(-2) + 2$
$= -6 + 2$
$= -4$

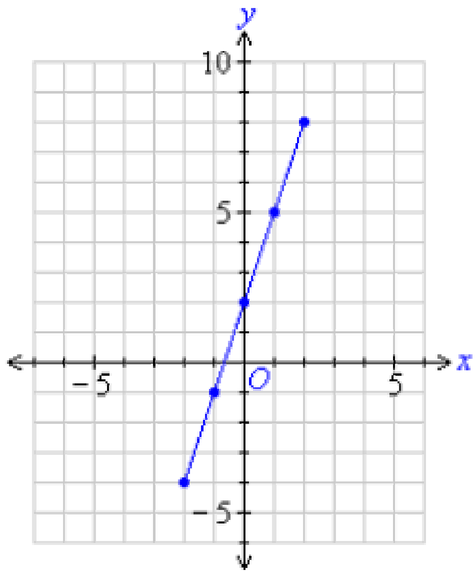
When $x = -1$, $y = 3(-1) + 2$
$= -3 + 2$
$= -1$

When $x = 0$, $y = 3 \times 0 + 2$
$= 0 + 2$
$= 2$

When $x = 1$, $y = 3 \times 1 + 2$
$= 3 + 2$
$= 5$

When $x = 2$, $y = 3 \times 2 + 2$
$= 6 + 2$
$= 8$

x	-2	-1	0	1	2
y	-4	-1	2	5	8



Example 7

Plot the graph of $y = –2x + 4$.

Solution:

$y = -2x + 4$

When $x = -2$, $y = -2(-2) + 4$
$= 4 + 4$
$= 8$

When $x = -1$, $y = -2(-1) + 4$
$= 2 + 4$
$= 6$

When $x = 0$, $y = -2 \times 0 + 4$

$= 0 + 4$

$= 4$

When $x = 1$, $y = -2(1) + 4$

$= -2 + 4$

$= 2$

When $x = 2$, $y = -2(2) + 4$

$= -4 + 4$

$= 0$

When $x = 3$, $y = -2(3) + 4$

$= -6 + 4$

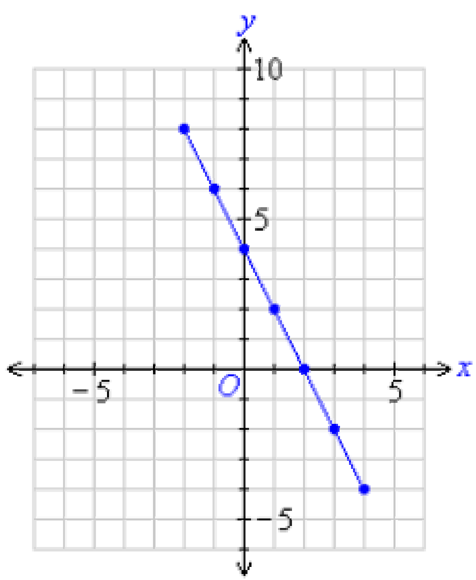
$= -2$

When $x = 4$, $y = -2(4) + 4$

$= -8 + 4$

$= -4$

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4



Trigonometry and Right Triangles

First of all, think of a trigonometry function as you would any general function. That is, a value goes in and a value comes out. If that does not seem quite clear, go see The Definition of a Function and What is f(x)?

The names of the three primary trigonometry functions are:

sine
cosine
tangent

These are abbreviated this way:

sine.....sin
cosine.....cos
tangent.....tan

So, instead of writing f(x) , we will write:

sin(x)
cos(x)
tan(x)

Often, in general mathematics notation the parentheses are dropped from the above examples. Therefore, the notation will often look like this:

sin x
cos x
tan x

In Zona Land we will keep the parentheses.

Now, as usual, the input value is x. This input value usually represents an angle. For the sine function, when the input value is 30 degrees, the output value is 0.5. We would write that statement this way:

$$0.5 = \sin(30\,°)$$

Below is a listing of several popular input and output values for the three main trigonometry functions. You do not have work at memorizing this table. After you use trigonometry for a while, these values will be remembered quite easily.

0.0000 = sin(0 °)	1.0000 = cos(0 °)	0.0000 = tan(0 °)
0.5000 = sin(30 °)	0.8660 = cos(30 °)	0.5773 = tan(30 °)
0.7071 = sin(45 °)	0.7071 = cos(45 °)	1.000 = tan(45 °)
0.8660 = sin(60 °)	0.5000 = cos(60 °)	1.7320 = tan(60 °)
1.0000 = sin(90 °)	0.0000 = cos(90 °)	+infinity = tan(90 °)

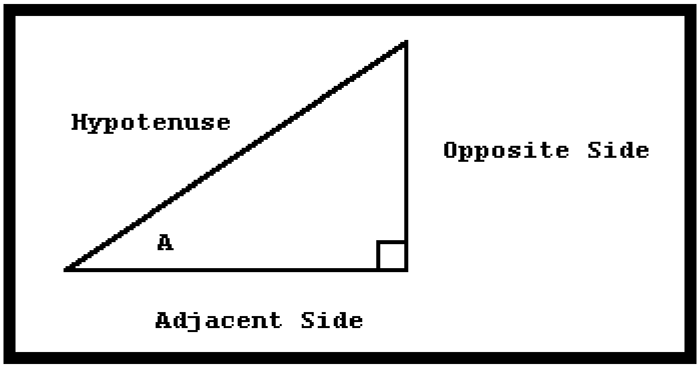
At this point our central issues will revolve around these questions:

Where do these numbers come from?
What do these numbers mean?
Why, for example, is the cosine of 30 degrees equal to 0.8660?

The input value for these trigonometric functions is an angle. That angle could be measured in degrees or radians. Here we will consider only input angles measured in degrees from 0 degrees to 90 degrees. This input value appears within the parentheses throughout the above table.

The output value for these trigonometric functions is a pure number. That is, it has no unit. This output value appears to the left of the equal sign throughout the above table.

There are several ways to understand why a certain input angle produces a certain output value. At first, the most important manner of understanding this is tied to right triangles. All of the trigonometric values for angles between 0 degrees and 90 degrees can be understood by considering this diagram:



We will be concerned angle A. Notice that the sides of the triangle are labeled appropriately "opposite side" and "adjacent side" relative to angle A. The hypotenuse is not considered opposite or adjacent to the angle A.

We will also be concerned with length of the three sides. For this discussion we will call the "length of the opposite side" simply the "opposite". Similarly, the other two lengths will be called "adjacent" and "hypotenuse".

The value for the sine of angle A is defined as the value that you get when you divide the opposite side by the hypotenuse. This can be written:

$$\sin(A) = \text{opposite} / \text{hypotenuse}$$

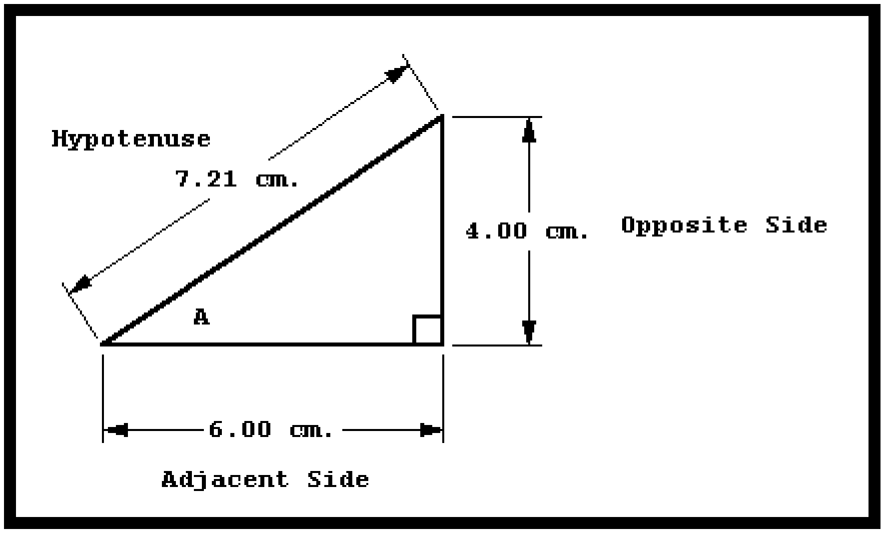
Or simply:

$$\sin(A) = \text{opp} / \text{hyp}$$

Or, even more simply:

$$\sin(A) = o / h$$

Suppose we measure the lengths of the sides of this triangle. Here are some realistic values:



This would mean that:

$$\sin(A) = \text{opposite} / \text{hypotenuse} = 4.00 \text{ cm} / 7.21 \text{ cm} = 0.5548$$

Or simply:

$$\sin(A) = 0.5548$$

Now for the other two trig functions.

The value for the cosine of angle A is defined as the value that you get when you divide the adjacent side by the hypotenuse. This can be written:

$$\cos(A) = \text{adjacent} / \text{hypotenuse}$$

Or:

$$\cos(A) = \text{adj} / \text{hyp}$$

Or:

$$\cos(A) = a / h$$

Using the above measured triangle, this would mean that:

$$\cos(A) = \text{adjacent} / \text{hypotenuse} = 6.00 \text{ cm} / 7.21 \text{ cm} = 0.8322$$

Or simply:

$$\cos(A) = 0.8322$$

The value for the tangent of angle A is defined as the value that you get when you divide the opposite side by the adjacent side. This can be written:

$$\tan(A) = \text{opposite} / \text{adjacent}$$

Or:

$$\tan(A) = \text{opp} / \text{adj}$$

Or:

$$\tan(A) = o / a$$

Using the above measured triangle, this would mean that:

$$\tan(A) = \text{opposite} / \text{adjacent} = 4.00 \text{ cm} / 6.00 \text{ cm} = 0.6667$$

Or simply:

$$\tan(A) = 0.6667$$

The angle A in the above triangle is actually very close to 33.7 degrees. So, we would say:

$$0.5548 = \sin(33.7°)$$
$$0.8322 = \cos(33.7°)$$
$$0.6667 = \tan(33.7°)$$

So, suppose that you wanted to know the trigonometry values for 47.5 degrees? You could carefully draw a right triangle using a ruler and protractor that had an angle equal to 47.5 degrees in the position of angle A. Then, you could carefully measure the sides. Lastly you could divide the appropriate sides to find the values for the three trigonometric functions. You would find that:

$$0.7373 = \sin(47.5°)$$
$$0.6755 = \cos(47.5°)$$
$$1.0913 = \tan(47.5°)$$

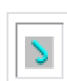
Someone has already done this, in a way, for all the possible angles. All the input angles and output values are listed in tables called trig tables. They look like this:

Angle	sin	cos	tan
0.0	0.0000	1.0000	0.0000
0.5	0.0087	0.9999	0.0087
1.0	0.0174	0.9998	0.0174

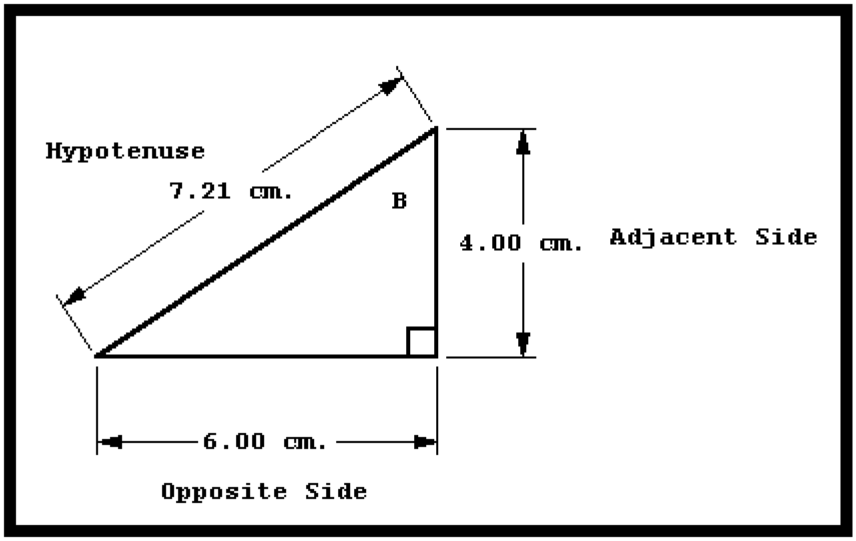
And so on...

These let you look up the trigonometric values for any angle. Calculators and computers, of course, will let you do the same.

Here is a demonstration that shows you these trig calculations for several angles. Use the slider to adjust the size of the angle. Notice how the values are calculated for each trig function depending upon the lengths of the sides of the triangle.



Below is again the triangle from the above diagrams, except now the other acute angle, B, is marked. Also marked are the sides that are adjacent and opposite to angle B.



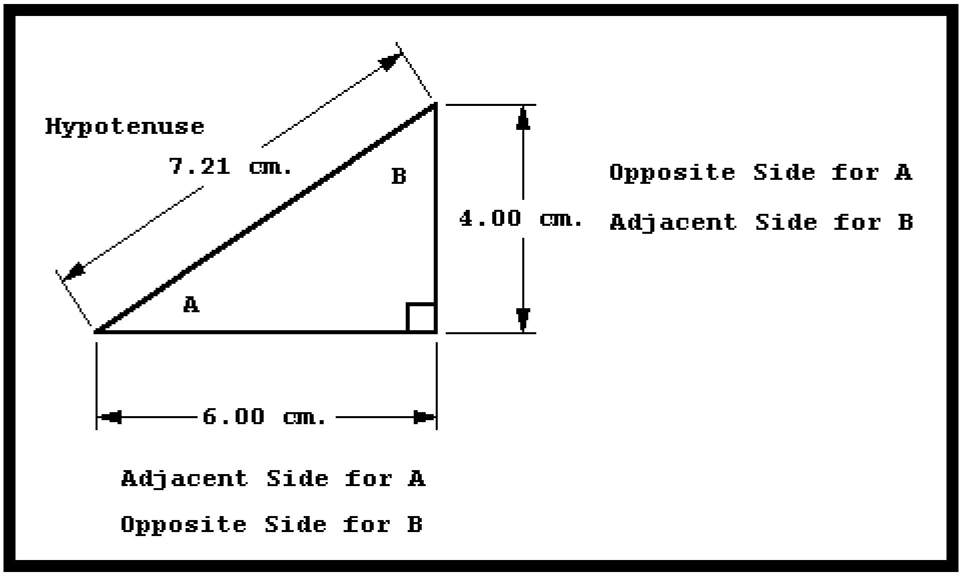
Here are the three trig functions for angle B:

sin(B) = opposite / hypotenuse = 6.00 cm / 7.21 cm = 0.8322
cos(B) = adjacent / hypotenuse = 4.00 cm / 7.21 cm = 0.5548
tan(B) = opposite / adjacent = 6.00 cm / 4.00 cm = 1.5000

If you look carefully you will notice that the sine of angle B is the same value we calculated above for the cosine of angle A. You should also notice that the cosine of angle B is equal to previous calculation for the sine of angle A.

This is because the opposite side for angle B is the adjacent side for angle A, and because the adjacent side for angle B is the opposite side for angle A.

This is demonstrated in the following diagram:



We could summarize this relationship this way:

sin(A) = A's opposite / hypotenuse = 4.00 cm / 7.21 cm = 0.5548
cos(B) = B's adjacent / hypotenuse = 4.00 cm / 7.21 cm = 0.5548
cos(A) = A's adjacent / hypotenuse = 6.00 cm / 7.21 cm = 0.8322
sin(B) = B's opposite / hypotenuse = 6.00 cm / 7.21 cm = 0.8322

Now, angle A and B form a pair of complementary angles. That is, their measurements add up to 90 degrees. This is because the measurement of the interior angles for any triangle must sum to 180 degrees, and in this triangle 90 of those degrees are taken up by the right angle, so that leaves 90 degrees remaining from the total of 180 to be split up between angle A and B.

So, here we notice that the sine of an angle is equal to the cosine of its complement, and that the cosine of an angle is equal to the sine of its complement.

Also, we will take note of the relationship between the tangents of the complementary angles A and B. The tangent of angle A is equal to the reciprocal, or inverse, of the tangent of angle B, and, likewise, the tangent of angle B is equal to the reciprocal of the tangent of its complement, angle A. This is summarized in the following table:

tan(A) = A's opposite / A's adjacent = 4.00 cm / 6.00 cm = 0.6667 tan(B) = B's opposite / B's adjacent = 6.00 cm / 4.00 cm = 1.5000

Here is an easy way to remember these relationships for trig functions and the right triangle. Just write down this mnemonic:

SOH - CAH - TOA

It is pronounced "so - ka - toe - ah".

The SOH stands for "Sine of an angle is Opposite over Hypotenuse."

The CAH stands for "Cosine of an angle is Adjacent over Hypotenuse."

The TOA stands for "Tangent of an angle is Opposite over Adjacent."
