ID: Solutions Date: 3/9/25

MARVLS: Quantum Computing - Introduction Bits, Qubits, Spin, and Gates

Materials

- Smartphone
- Merge Cube
- MARVLS: Quantum Computing App

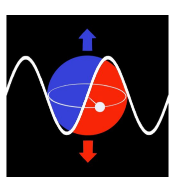
App Store Google Play

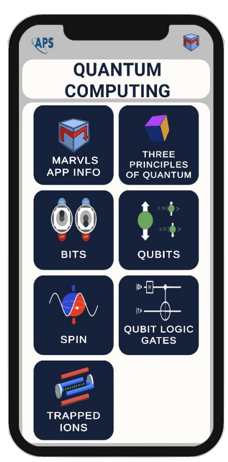
Quantum Computing

Bits, Qubits, Spin, and Gates

MARVLS Navigation

Open the Quantum Computing App





Name: Solutions Date: ____________
Module 1: Bits vs qubits
Lesson 1: What are bits and how are they used?

Learning Objective	I can understand and describe a bit and how bits are combined to store numbers and data.

What's the difference between binary and decimal?

In the normal decimal number system (i.e. 0, 1, 2...9), a single digit can have a value between 0 and 9. In the binary number system the single digit can only have a value of 0 and 1. If the normal number system is called base 10 then what would the binary number system be called? Base ______ (fill in the blank)

A digit in the binary system is called a bit. In the MARVLS quantum computing App, try out the Binary Numbers ON-Off Scene to see a bit in augmented reality. See how to access the scene at the bottom of the page. Click on the ON and OFF buttons to see how the value of a bit changes.

A bit can only have a value of 0 or 1.

Counting with binary numbers

When a digit can only have the values 0 and 1, we call it a binary number. In the table below, 16 binary numbers in order are given as 4 digit values with their decimal value also shown.

Binary number	Decimal number
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary number	Decimal number
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

Go into the **MARVLS Quantum Computing App**, try out the **Binary Counting Scene** of 4 bits in augmented reality. See how to access the scene at the bottom of the page. Click on the Add One Button to add 1 each time to see how the binary number changes.

Q1. As the numbers increase, what do you notice about how often the last digit changes (the right-most bit)? Fill in the blank.

The right-most bit changes from 0 to 1 or 1 to 0 every time "Add One" is pressed.

Q2. What change do you notice in the second-to-last bit? The third-to-last bit? The first bit? Fill in the blank.

The second-to-last bit changes every other time the button is pressed.

Q3. How is it like adding in the decimal system when you need to go from 9 to 10? Fill in the blank.

Adding one to go from 9 to 10 is like adding one to go from 1 to 10 in binary.

MARVLS Navigation

Open the Quantum Computing App → Choose BITS → Choose ON-OFF or Binary Counting → ON-OFF / BINARY COUNTING

Name: Solutions Date: ______________
Module 1 Bits vs Qubits
Lesson 2: How to calculate the decimal value of a binary number?

Learning Objective	I can calculate the decimal value of a binary number.

What's the math trick?

Each digit in a binary number equals a decimal number based upon the location of the digit in the number. From the table on the previous page, fill in the decimal value for each of these binary numbers.

0001 = 1 0010 = 2 0100 = 4 1000 = 8

For any 4 digit binary number, add the values you found above when the value of that digit is a 1. Here's an example, let's look at the number 1011.

1011 = 8 + 2 + 1 = 11

Notice that we didn't add 4 because the second digit is zero.

Try using this technique to find the decimal value of these numbers. Check your answers on the previous page.

1010 = 8+2 = 10 0101 = 4+1 = 5

0110 = 4+2 = 6 1111 = 8+4+2+1 = 15

What about more bits?

Based on the pattern you see above, what is the decimal value for this binary numbers? (hint: notice that for your four digit number, as you moved to the next digit, you multiplied the value by 2).

5-bits: 10000 = 16 6-bits: 100000 = 32

7-bits: 1000000 = 64 8-bits: 10000000 = 128

Here's an example of an 8-bit number. Let's look at the number 10101010.

10101010 = 128 + 32 + 8 + 2 = 170

Try finding the decimal value of these numbers. Check your answers on the previous page.

11011010 = 128+64+16+8+2 = 218 11111111 = 128+64+32+16+8+4+2+1 = 255

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Module 1: Bits vs Qubits

Lesson 3: What are qubits?

Learning Objective	I can understand and describe the differences between a bit and a quantum bit (qubit).

What's the difference?

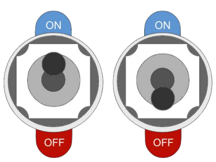
Unlike a classical bit, which can only be in one of two states (0 or 1), a qubit can exist in a linear combination of both states simultaneously. This principle is called **superposition**. This means that we can do calculations with both values at the same time. When we have a large number of bits, this means we can do many calculations at the same time.

How are they represented?

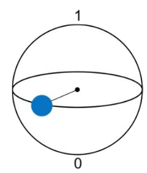
Mathematically, the value of a bit is 0 or 1. For a qubit, the states are written in "ket" notation |0> and |1>. This means that the qubit can be in either state until it is measured.

> The states of a qubit are |0> and |1>.
> Superposition means that the qubit is in both states at the same time.

Physically, bits are represented as an on-off switch. While qubits are represented by the Bloch sphere. The switches illustrate that bits only have values of 0 and 1. The Bloch sphere shows that the state of a qubit can be in between |0> and |1>. When the bob is on the equator that means that there's a 50% chance the state is |0> and a 50% chance that the state is |1>.



Bits represented as on-off switches

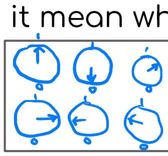


Qubits represented by the Bloch sphere

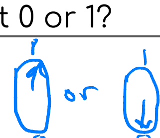
1. Open the **MARVLS Quantum Computing App** and view the **Qubit** Scene (see below).
2. Click the view in AR button on the bottom of the screen.
3. Click on the button super to see a single in a superposition of states |0> and |1>. Notice that the percentages show you the probability that a qubit is in either state. Click on the button measure to see that the state of the qubit is 0 or 1 when the qubit is measured. The percentages that you see when the qubit is measured shows the probability just before it was measured. Describe what you see when the qubit is in each state.

superposition state	measured state
The arrow in the Bloch sphere can be 0,1 or somewhere in between. The arrow direction is connected to the probability the qubit is in the zero or one state.	When the qubit is measured, the state of the qubit is zero or one only.

4. Next, open the Qubit vs Bit Scene (see below). Click on the change state button a few times.
5. What do you notice about the values for the bit as compared to the qubit values? What does it mean when the blue arrow isn't pointing at 0 or 1?



qubit A qubit can can have a value of 0 or 1 or something in between.

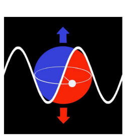


or

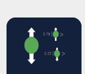
bit A bit can only be in state 0 or 1.

MARVLS Navigation

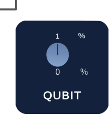
Open the Quantum Computing App

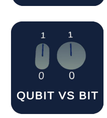


Choose QUBITS



Choose QUBIT and QUBIT vs BIT





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Module 1: Bits vs Qubit
Lesson 4: Qubits and entanglement

What is entanglement and how it is used in quantum computing?

So far we know that superposition is a superpower of qubits. Entanglement is another superpower. When these qubits become entangled, it's like they make a secret agreement: if you look at one, you instantly know something about the other. For example, if qubit A is measured to have a certain value, qubit B will have a value that is perfectly related to it—even if they're on opposite sides of the universe!

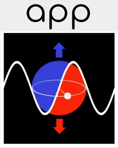
Here's a simple analogy: Suppose you and a friend each get a box, and inside each box is either a red ball or a blue ball. Before you open the boxes, you don't know what's inside. But if the boxes are "entangled," the moment you open yours and see a red ball, your friend's box will definitely have a blue ball (or vice versa). This happens instantly, no matter the distance between you!

In the quantum world, this isn't just a cool trick—it's a fundamental property. Entanglement is what makes quantum computing so powerful because it allows qubits to work together in ways classical bits can't!

Time to View Entangled Qubits in 3D AR!

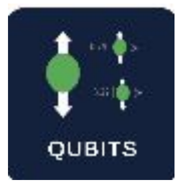
MARVLS Navigation

Open the app



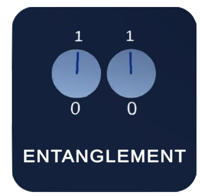
→ Choose

The qubit menu button

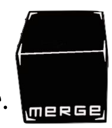


→ Choose

Entanglement



1. Open the **MARVLS Quantum Computing App** and view the **Entanglement** Scene (see below).
2. **Click** the view in AR button on the bottom of the screen.
3. **Point** your phone's camera toward the Merge Cube.
4. Change the cube's orientation until your cube matches the cartoon image of the cube.



5. Select Entangle at the bottom on the screen to open the bottom menu. Click on the blue Measure button. Try this a few times. Describe what you see.

When I press the measure button, the qubits are in states zero or one. Sometimes their states match and sometimes they don't.

6. Click on Super (short for superposition) to reset the qubits.
7. **Check** the entangled box. Then click on the blue Measure button. Try this a few times. Try this a few times. Describe what you see.

When I press the measure button the states of the qubits change but the measured states of both qubits always match with a value of zero or one.

8. Click on Super (short for superposition) to reset the qubits.
9. With the entangled box checked, **check** the opposite box. Then click on the blue Measure button. Try this a few times. Try this a few times. Describe what you see.

When I press the measure button the states of the qubits change but the measured states of both qubits never match. For example if the qubit on the left is zero, the qubit on the right will have a value of one and vice versa.

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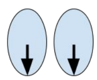
Module 1: Bits vs Qubits

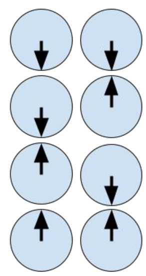
Lesson 5: Qubits and calculations

Why can qubits do more calculations than bits?

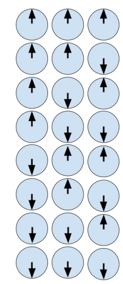
So far we know that **superposition** is a superpower of qubits. This superpower allows each qubit to be in a state of |0> or |1> at the same time. This means that if we have two qubits, there are 4 calculations that occur at the same time. The figure below shows a calculation using two classical bits (left) and two qubits (middle). Because the classical bits can only be in one state prior to the calculation, it can only do one calculation. However, because the qubits can be in a superposition of states, all 4 possible calculations occur at the same time. That means we can do 4 calculations 4 times faster! As we increase the number of bits, that number increases a lot! For example, as you'll see in the MARVLS App, if we have 3 bits, we can do 8 calculations at the same time (right). With 4 bit we can do 16 calculations at the same time. For each bit we add, we increase the number of calculations by a factor of 2!



Two classical bit calculation



Two qubit calculation

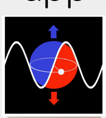


Three qubit calculation

Compare Calculations with Classical Bits and Qubits in 3D AR!

MARVLS Navigation

Open the app

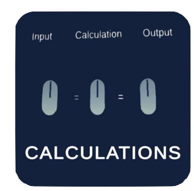


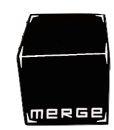
→ Choose

The qubit menu button

→ Choose

Entanglement





1. Open the **MARVLS Quantum Computing App** and view the **Calculations** Scene.
2. **Click** the view in AR button on the bottom of the screen.
3. **Point** your phone's camera toward the Merge Cube.
4. Change the cube's orientation until your cube matches the cartoon image of the cube.
5. You will see labels for input, calculation, and output. Below those labels you will see input bits, calculation bits, and output bits. You can increase the number of input bits to 3 and you can compare the differences between using classical bits and qubits.
6. Complete the table below based on what you see in the App for different numbers of bits and qubits. Increase the number of bits by pressing the add one button. Change the bit type by pressing the bit and qubit button. The App doesn't go beyond 3, so you'll need to figure it out!

# of Classical input bits	# of Classical Calculations	# of Classical Output Bits	# of Qubit input bits	# of Qubit Calculations	# of Qubit Output Bits
1	1	1	1	2	1
2	2	2	2	4	2
3	3	3	3	8	3
4	4	4	4	16	4
5	5	5	5	32	5
6	6	6	6	64	6
7	7	7	7	128	7
8	8	8	8	256	8

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Module 1: Bits vs Qubits

Lesson 6: Gates for classical bits and qubits are NOT the same!

Are gates used with bits the same as the qubit gates? NO!

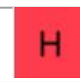
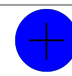
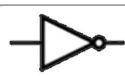
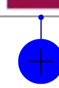
Gates used with classical bits make up most of the functionality of a classical computer. Transistors that make up computer processors are comprised of a series of gates including AND, OR, NOT, NAND, and XOR gates. Gates allow bits to be manipulated. The outputs of classical gates are listed in truth tables and they include the inputs and output values for each gate.

What's so special about quantum computing gates?

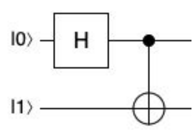
We've learned that **superposition** is a superpower of qubits. Quantum computing gates allow the qubits to be controlled and set to certain values prior to a calculation. To take full advantage of qubits, gates allow the qubits to start out in a state of 0, 1, or a superposition of states, |0> or |1>.

Gates can be arranged into algorithms to aid in quantum calculations. Gates typically do four things. The Hadamard gate puts qubits into a superposition state (equally likely to be in a |0> or |1> state at the same time). A Phase Rotation also puts a qubit into a superposition of |0> or |1> state but one state may be more likely than the other. A NOT gate changes the state from |0> to |1> or vice versa. Another important gate is the control gate where the state of one qubit is controlled by the state of the other.

Quantum computing programmers create circuits of these gates. The table below lists some gates (both classical and quantum) and their circuit components.

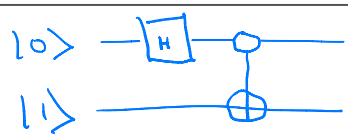
Classical Gates	Circuit Component	Quantum Gates	Circuit Component
AND		Hadamard gate	
OR		NOT gate	
NOT		Ry gate	
NAND		CNOT gate	

A simple 2-qubit circuit is shown below. This circuit includes a hadamard gate on the q_0 qubit and a CNOT gate on the q_1 qubit. This quantum circuit creates two qubits that are in a superposition of both |0> and |1> and these qubits are entangled. This means that if the value of q_0 is measured to be 1, then q_1 will also have a value of 1.



A few things you need to know to understand how the circuit works: (1) both q_0 and q_1 start out with values of zero. The Hadamard gate causes q_0 to be in a superposition of the |0> state and the |1> state. This means that the value of q_0 is equally likely to have a value of 0 or 1. The CNOT gate flips the state of q_1 ONLY if the value of q_0 is 1. So if the value of q_0 is zero, the CNOT gate does not change the value of q_1 and both qubits equal 0. If the value of q_0 is 1, the CNOT gate DOES flip the value of q_1 from 0 to 1 and now both q_0 and q_1 are 1. These qubits are now entangled.

Can you create a circuit below where if the value of q_0 is measured to be 0, then q_1 will be measured to be 1 and if q_0 is 1, then, q_1 will have a value of 0?



Change the CNOT gate to use an anti-control

Instead of this  use this gate 

This changes the value of q_1 from 0 to 1 only when $q_0 = 0$.

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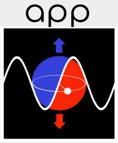
Module 2: Using Electron Spin States as a Qubit

Learning Objective	I can understand that a spin down electron is the |0> state and a spin up electron is the |1> state.

MARVLS Navigation

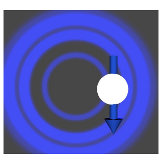
Open the app	Choose Spin		Choose Electron Spin	
	Spin		Spin	

Do electrons spin?

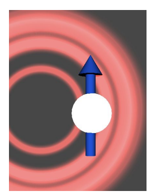
As a matter of fact, no, not really. The spin of the electron is related to the dipole moment of the electron and magnetic field surrounding it. Because it has a magnetic dipole moment, an electron can rotate due to its interaction with a magnetic field. The spin of an electron is a terrific candidate for a qubit for two reasons. First, electron spin is only two states, up or down. Second, laser pulses can add energy to an electron in the down state and cause it to rotate into the up state! This energy is the only energy that can cause the electron to switch so it's possible to control the spin of the electron.

Quantum scientists take advantage of of this energy difference between spin up and spin down states by sending an electromagnetic wave or a photon of light that's tuned to this energy. In this way they can manipulate the state of qubits.

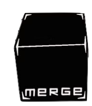
To learn more about how scientists manipulate the spin of electrons, we will illuminate an electron with a laser source. In order for the electron to change states, we need select a very specific frequency of the laser source.



Higher energy



Lower energy



Once we have the frequency, the electron will absorb just the right amount of energy to change states. In our example, we show three transitions between states, from the down to the up state (0 to 1), from the up to the down state (1 to zero), and from the up state to a superposition state.

1. Open the **MARVLS Quantum Computing App, choose Spin, and then** choose **Electron Spin**. Then click the icon with the image and red camera.
2. To read the instructions, at the bottom of the screen, click on How to Play.
3. To get started, click on Buttons at the bottom of the screen.
4. Point the camera of your phone at the cube in the orientation shown to the right.
5. Check the laser checkbox to turn on the laser. laser ☐
6. Check the d to u box to select the down to up state transition. u to d ☐
7. Use the slider to find the exact frequency for this transition. You'll know your close when the electron glows the same color as the laser pulse. Watch closely to see the arrow change.

1 ——— 5
frequency (GHz)

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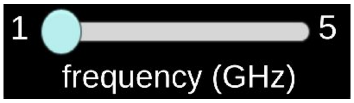
Module 2: Using Electron Spin States as a Qubit - con't

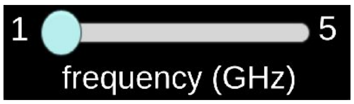
1. Next, to see the up to down state transition, check the u to d box. 

2. Use the slider to find the exact frequency for this transition. You'll know your close when the electron glows the same color as the laser pulse. Watch closely to see the arrow change.



3. Next, to see the superposition transition, check the super box. 

4. Use the slider to find the exact frequency for this transition. You'll know your close when the electron glows and the pulse button appears. You will see the arrow change from pointing down to pointing up.



5. Press the pulse button to see what happens! 

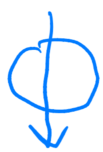
6. To reset and try again, uncheck to the boxes and start again.

For each of the three examples below,

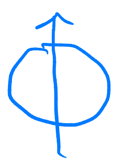
1. draw the electron-arrow pair before and after the transition. Make sure to notice which direction the arrow started in and ended in.
2. Note which case needed a higher energy and how did you know?
3. For the last example, draw what you see and describe what's needed to put the electron qubit into a state of superposition. How do you know it's in superposition?

Down to up transition

before



after

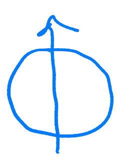


Higher energy

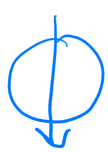
Purple has a higher frequency and energy as compared to red.

Up to down transition

before



after

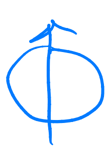


Lower energy

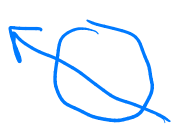
red energy < purple energy

Superposition transition

before



after 3 pulses



Superposition needs a specific energy and a certain number of pulses to rotate the spin.

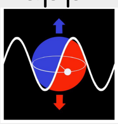
It is in superposition because the arrow is not pointing to zero or one, but somewhere in between.

Name: Solutions **Date:** ________
Module 3: Qubit Logic Gates

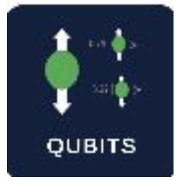
Learning Objective	I can understand the states of qubits and how gates are used to change states and prepare qubits to be used in algorithms.

MARVLS Navigation

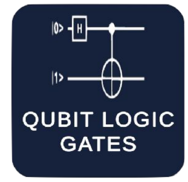
Open the app



Choose The qubit menu button



Choose Qubit Logic Gates



let's make these qubits DO Something!

In this lesson we're going to make some quantum circuits. We'll practice with the MARVLS App and connect the pictures of the circuits with what's happening to the qubits in augmented reality. We'll represent each qubit with a Bloch sphere and we'll observe what the beginning states of the qubits and what each subsequent gate does to each qubit.

In the circuits you'll be making, each qubit has a line next to it that is it's register. You'll drag gates onto that line. For these examples, the gates are expected to be pretty close to the left side. So if the gate isn't attaching when you drag it to the line, try dropping it closer to the qubit label.

each

1. Open the MARVLS Quantum Computing App and view the Qubit Logic Gates Scene.
2. Click the view in AR button on the bottom of the screen.
3. Point your phone's camera toward the Merge Cube.
4. Change the cube's orientation until your cube matches the cartoon image of the cube.

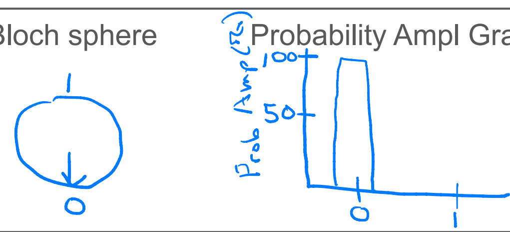
Let's test out three different gates and see what they do to a **single qubit**.

5. Open the 1 qubit menu at the bottom of the scene
6. Drag the NOT gate onto the register for q_0

Draw and describe what changes. Include a drawing of the sphere before and after as well as the graphs before and after.

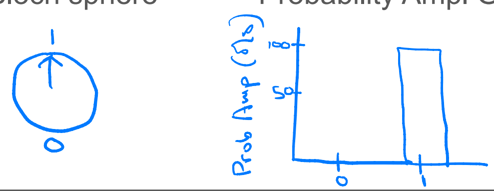
BEFORE

Bloch sphere Probability Ampl Graph



AFTER

Bloch sphere Probability Ampl Graph



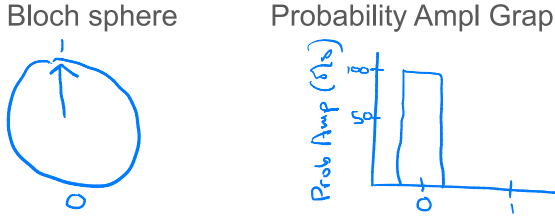
Describe the change: Before the gate is applied, $q_0 = 0$. After the gate is applied $q_0 = 1$.

7. Remove the NOT gate and drag the Hadamard gate H onto the register for q_0

Draw and describe what changes. Include a drawing of the sphere before and after as well as the graphs before and after.

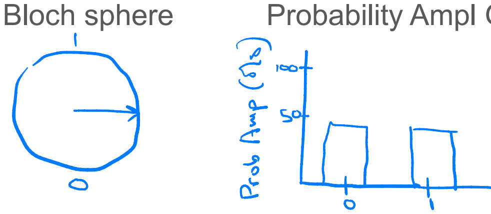
BEFORE

Bloch sphere Probability Ampl Graph



AFTER

Bloch sphere Probability Ampl Graph



Describe the change: Before the gate is applied $q_0 = 0$. After the gate q_0 is in an equal superposition of zero and one.

Name: Solutions **Date:**

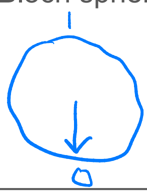
Module 3: Qubit Logic Gates con't

8. Remove the Hadamard gate and drag the Phase Rotation gate (Ry) onto the register for q_0 Draw and describe what changes. Include a drawing of the sphere before and after as well as the graphs before and after.

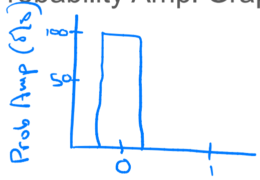
For angle of 45°

BEFORE

Bloch sphere

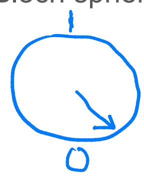


Probability Ampl Graph

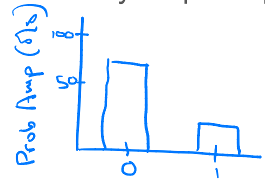


AFTER

Bloch sphere



Probability Ampl Graph



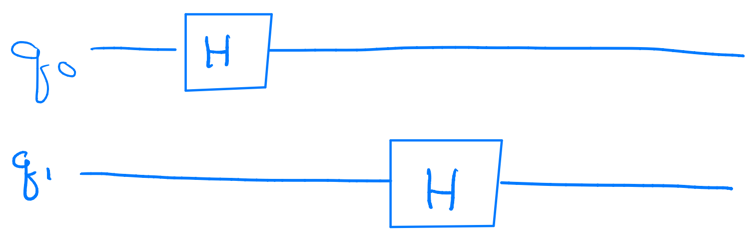
Describe the change: Before the gate, $q_0 = 0$. After the gate q_0 is in a superposition of zero and one with a higher probability of being in the zero state.

2 Qubit Circuits

Let's test out a few circuit and see what they do to **two qubits**.

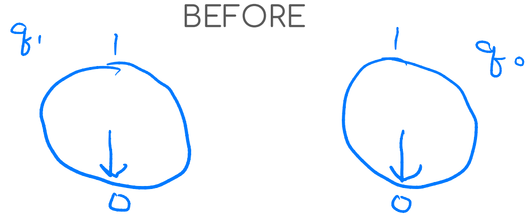
1. Close the 1 qubit menu and open the 2 qubit menu at the bottom of the scene.
2. Drag the Hadamard gate (H) onto the register for q_0 and another one for q_1.

Draw the **circuit** that you've made.

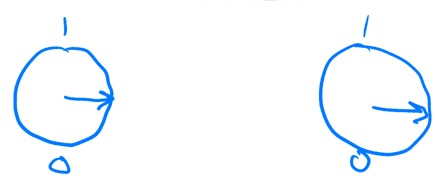


Draw the **Bloch spheres** before and after to show the states.

BEFORE

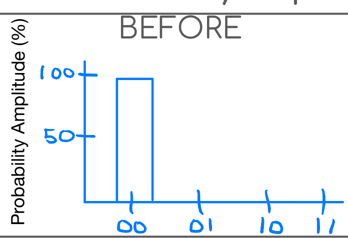


AFTER

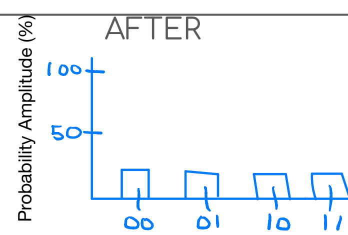


Draw the **Probability Amplitude Graphs** before and after to show the states.

BEFORE



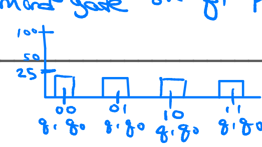
AFTER



Describe how the circuit works and what's happening to the qubits.

Before the gates, $q_0 = 0$ and $q_1 = 0$. The Hadamard gate on q_0 puts it into superposition with equal probability of zero and one. The second Hadamard gate on q_1 puts it into superposition with equal probabity of zero and one.

The graph shows probabilities



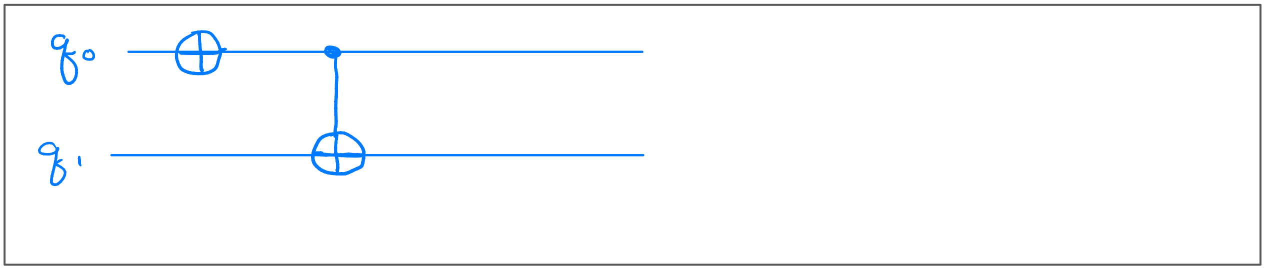
Name: Solutions Date:

Module 3: Qubit Logic Gates con't

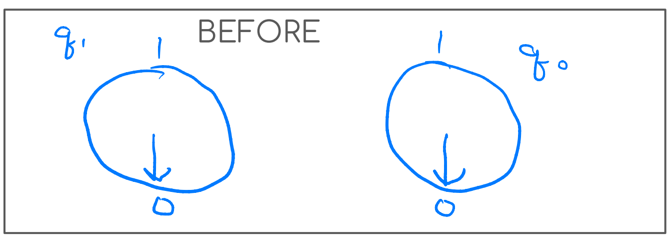
Control Qubits

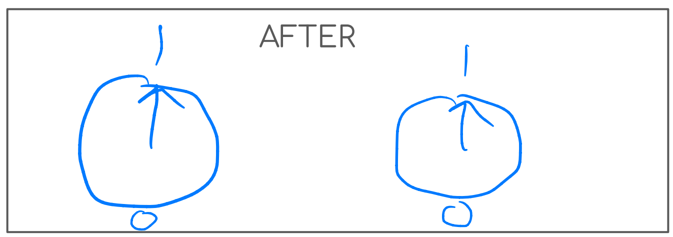
1. Remove all gates from the registers to reset.
2. Drag the Not gate ⊕ onto the register for q_0.
3. Drag the CNOT gate ⊕ onto the register for q_1.

Draw the **circuit** that you've made.

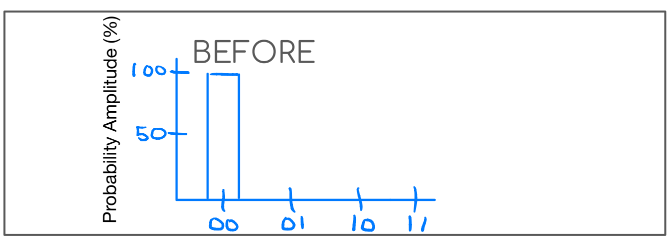


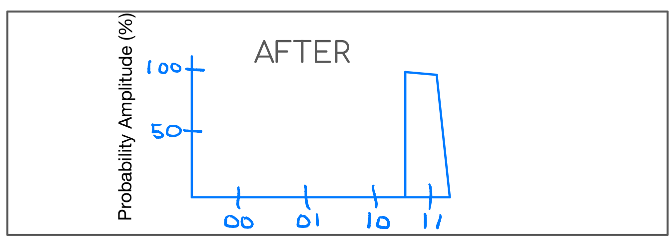
Draw the **Bloch spheres** before and after to show the states.





Draw the **Probability Amplitude Graphs** before and after to show the states.





Describe how the circuit works and what's happening to the qubits.

Before the gates, $q_0=0$, $q_1=0$. The NOT gate on q_0 changes its state from 0 to 1. The control NOT (C-NOT) on q_1 changes q_1 from 0 to 1 only because $q_0=1$. Both qubits are in the state of 1.

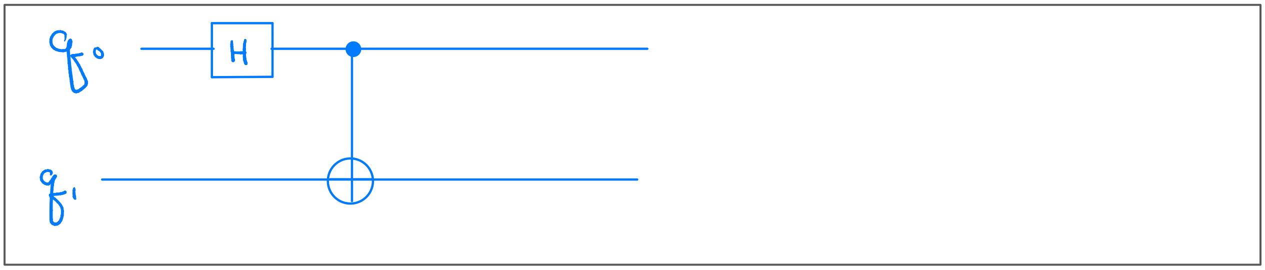
Name: Solutions   Date: ______________

Module 3: Qubit Logic Gates con't

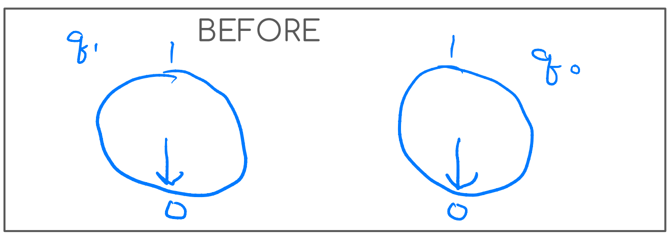
Entangling 2 Qubits

1. Remove all gates from the registers to reset.
2. Drag the Hadamard gate H onto the register for q_0.
3. Drag the CNOT gate on the register for q_1.

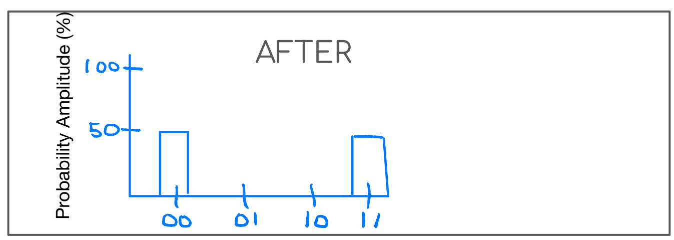
Draw the **circuit** that you've made.



Draw the **Bloch spheres** before and after to show the states.



Draw the **Probability Amplitude Graphs** before and after to show the states.



Describe how the circuit works and what's happening to the qubits.

Before the gates, $q_0 = 0$ and $q_1 = 0$. The Hadamard gate puts q_0 into an equal superposition of 0 and 1. The C-NOT gate applied to q_1 sets q_1 to 1 only when $q_0 = 1$. This means that q_0 and q_1 are entangled. When $q_0 = 0$, $q_1 = 0$ and when $q_0 = 1$, $q_1 = 1$.